

Creative Telescoping

4.1 Univariate D-finite Functions

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Recent Trends in Computer Algebra
Special Week @ Institut Henri Poincaré



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Many Functions are D-Finite

ArcCsc, KelvinBei, HypergeometricPFQ, ExpIntegralE, ArcTanh, HankelH2, AngerJ, JacobiP, ChebyshevT, AiryBi, AiryAi, Sinc, CosIntegral, ArcSech, SphericalBesselY, Sin, WhittakerW, SphericalHankelH2, HermiteH, ExpIntegralEi, Beta, AiryBiPrime, SphericalBesselJ, ParabolicCylinderD, Erfc, EllipticK, Cos, Hypergeometric2F1, Erf, KelvinKer, BetaRegularized, HypergeometricPFQRegularized, Log, BesselY, Cosh, ArcSinh, CoshIntegral, ArcTan, ArcCoth, LegendreP, LaguerreL, EllipticE, SinhIntegral, Sinh, SphericalHankelH1, ArcSin, AiryAiPrime, EllipticThetaPrime, Root, AppellF1, FresnelC, LegendreQ, ChebyshevU, GammaRegularized, Erfi, BesselI, HypergeometricU, KelvinKei, Exp, ArcCot, Hypergeometric2F1Regularized, ArcSec, Hypergeometric0F1, EllipticPi, GegenbauerC, ArcCos, WeberE, FresnelS, EllipticF, ArcCosh, HankelH1, Sqrt, BesselK, BesselJ, Hypergeometric1F1Regularized, StruveL, KelvinBer, StruveH, WhittakerM, ArcCsch, Hypergeometric1F1, SinIntegral, ...

Quiz: Who is D-Finite?

$$\operatorname{erf}(\sqrt{x+1})^2 + \exp(\sqrt{x+1})^2$$

$$\left((\sinh(x))^2 + (\sin(x))^{-2} \right) \cdot \left((\cosh(x))^2 + (\cos(x))^{-2} \right)$$

$$\frac{\log(\sqrt{1-x^2})}{\exp(\sqrt{1-x^2})}$$

$$\arctan(e^x)$$

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Quiz: Who is D-Finite?

$$\operatorname{erf}(\sqrt{x+1})^2 + \exp(\sqrt{x+1})^2 \quad \checkmark$$

$$\left((\sinh(x))^2 + (\sin(x))^{-2} \right) \cdot \left((\cosh(x))^2 + (\cos(x))^{-2} \right) \quad \times$$

$$\frac{\log(\sqrt{1-x^2})}{\exp(\sqrt{1-x^2})}$$

$$\arctan(e^x)$$

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$$\frac{\log(\sqrt{1-x^2})}{\exp(\sqrt{1-x^2})} \quad \checkmark$$

$$\arctan(e^x)$$

Quiz: Who is D-Finite?

$$\operatorname{erf}(\sqrt{x+1})^2 + \exp(\sqrt{x+1})^2 \quad \checkmark$$

$$\left((\sinh(x))^2 + (\sin(x))^{-2} \right) \cdot \left((\cosh(x))^2 + (\cos(x))^{-2} \right) \quad \times$$

$$\frac{\log(\sqrt{1-x^2})}{\exp(\sqrt{1-x^2})} \quad \checkmark$$

$$\arctan(e^x) \quad \times$$

Many Sequences are P-Recursive

Multinomial, KelvinBei, HypergeometricPFQ, HarmonicNumber, HankelH2, CatalanNumber, AngerJ, JacobiP, ChebyshevT, SphericalBesselY, WhittakerW, Gamma, Subfactorial, BesselJ, Pochhammer, SphericalHankelH2, Fibonacci, HermiteH, Beta, SphericalBesselJ, Tribonacci, StruveL, ParabolicCylinderD, Hypergeometric2F1, BesselK, BetaRegularized, KelvinKer, PolyGamma, HypergeometricPFQRegularized, SchröderNumber, SphericalHankelH1, LegendreP, LaguerreL, DelannoyNumber, BetaRegularized, AppellF1, LegendreQ, Binomial, ChebyshevU, GammaRegularized, Bessell, HypergeometricU, KelvinKei, Factorial, Hypergeometric2F1Regularized, GegenbauerC, KelvinBer, WeberE, HankelH1, Hypergeometric1F1Regularized, StruveH, WhittakerM, Hypergeometric0F1, Factorial2, Hypergeometric1F1, LucasL, MotzkinNumber, BesselY, ...