In[1]:=
```
<< RISC`fastZeil`
```

Fast Zeilberger Package version 3.61
written by Peter Paule, Markus Schorn, and Axel Riese

In[2]:=
```
(* Example from Shaoshi's lecture *)
Gosper[Binomial[m, k] / Binomial[n, k], k]
```

Out[2]= $\left\{\frac{\text{Binomial}[m, k]}{\text{Binomial}[n, k]} == \Delta_k\left[\frac{(1-k+n)\,\text{Binomial}[m, k]}{(-1+m-n)\,\text{Binomial}[n, k]}\right]\right\}$

In[3]:=
```
(* Binomial coefficient is not Gosper-summable *)
Gosper[Binomial[n, k], k]
```

Out[3]= $\{\}$

In[4]:=
```
Gosper[(2 k - n - 1) / (n - k + 1) * Binomial[n, k], k]
```

Out[4]= $\left\{-(((-1+2k-n)\,\text{Binomial}[n, k]) / (-1+k-n)) == \Delta_k\left[\frac{k\,\text{Binomial}[n, k]}{-1+k-n}\right]\right\}$

In[5]:=
```
Zb[Binomial[n, k], k, n]
```

Out[5]= $\{2\,F[k, n] - F[k, 1+n] == \Delta_k[F[k, n]\,R[k, n]]\}$

In[6]:=
```
show[R]
```

Out[6]= $\frac{k}{1-k+n}$

In[7]:=
```
Zb[(-1)^k * Binomial[2 n, n + k]^2, k, n]
```

Out[7]= $\{-2\,(1+2n)\,F[k, n] + (1+n)\,F[k, 1+n] == \Delta_k[F[k, n]\,R[k, n]]\}$

In[8]:=
```
Zb[(-1)^k * Binomial[2 n, n + k]^3, k, n]
```

Out[8]= $\{6\,(1+3n)\,(2+3n)\,F[k, n] - 2\,(1+n)^2\,F[k, 1+n] == \Delta_k[F[k, n]\,R[k, n]]\}$

In[9]:=
```
Zb[Binomial[n, k]^2 * Binomial[n + k, k]^2, k, n]
```

Out[9]= $\{(1+n)^3\,F[k, n] - (3+2n)\,(39+51n+17n^2)\,F[k, 1+n] + (2+n)^3\,F[k, 2+n] == \Delta_k[F[k, n]\,R[k, n]]\}$

In[10]:=
```
Zb[(-1)^k * Binomial[n, k] * Binomial[2 * k, n], k, n]
```

Out[10]= $\{-2\,(1+n)\,F[k, n] + (-1-n)\,F[k, 1+n] == \Delta_k[F[k, n]\,R[k, n]]\}$

In[11]:=
```
Zb[(-1)^k * Binomial[n, k] * Binomial[3 * k, n], k, n]
```

Out[11]= $\{9\,(1+n)\,(2+n)\,F[k, n] + 3\,(2+n)\,(7+5n)\,F[k, 1+n] + 2\,(2+n)\,(3+2n)\,F[k, 2+n] == \Delta_k[F[k, n]\,R[k, n]]\}$

In[12]:= **Zb[(-1)^k*Binomial[n, k]*Binomial[4*k, n], k, n]**

Out[12]= $\{-64\ (1+n)\ (2+n)\ (3+n)\ (7+3\ n)\ F[k,n] - 16\ (2+n)\ (3+n)\ (107+125\ n+33\ n^2)\ F[k,1+n] -$
$4\ (3+n)\ (4+3\ n)\ (218+180\ n+37\ n^2)\ F[k,2+n] -$
$3\ (3+n)\ (4+3\ n)\ (7+3\ n)\ (8+3\ n)\ F[k,3+n] == \Delta_k[F[k,n]\ R[k,n]]\}$