Creative Telescoping

2.3 Programming of Sister Celine's Method

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Recent Trends in Computer Algebra
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7. Sum over the k-free recurrences and return the result.

Examples

$$\sum_{k=0}^{n} \binom{n}{k} = 2^n$$

$$\sum_{k=0}^{n} \binom{n}{k}^2 = \binom{2n}{n}$$

$$\sum_{k=-n}^{n} (-1)^k \binom{2n}{n+k}^2 = \frac{(2n)!}{(n!)^2}$$

$$\sum_{k=0}^{n} \binom{n}{k}^2 \binom{n+k}{k}^2 \quad \rightsquigarrow \quad \text{second-order recurrence}$$

$$\sum_{k} (-1)^k \binom{l+m}{l+k} \binom{m+n}{m+k} \binom{n+l}{n+k} = \frac{(l+m+n)!}{l!\, m!\, n!}$$