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A Geometric Approach to the Coverage Measure of the Area Explored by a Robot Certified and Symbolic-Numeric Computation Workshop

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May 23, 2023





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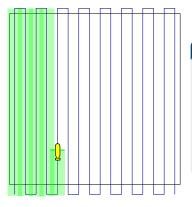
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Case of Study



Context

- Unknown environment,
- area covering mission,
- revisiting,
- region avoidance,
- line-sweep exploration.





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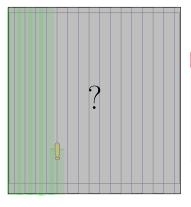
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Objectives

Using only proprioceptive data, to estimate:

- Explored area
- Number of views (coverage measure)





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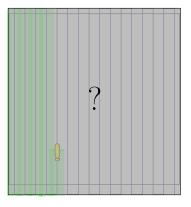
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Applications:

- Assess area-covering missions,
- plan other missions to fill possible gaps,
- assess revisiting missions,
- optimal trajectory planning,
- localization in homogeneous environments.





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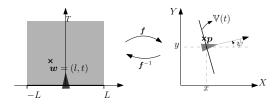
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Visible Area

 $\mathbb{V}:[0,T]\to\mathcal{P}(\mathbb{R}^2)$







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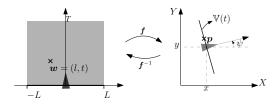
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Visible Area

 $\mathbb{V}:[0,T]\to\mathcal{P}(\mathbb{R}^2)$



Waterfall Space and Sweep Function

$$W = [-L, L] \times [0, T]$$
$$f: W \to \mathbb{R}^2$$



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Robot's Trajectory

- $\boldsymbol{x}: [0, T] \rightarrow \mathbb{R}^2$,
- \boldsymbol{x} is differentiable in [0, T].

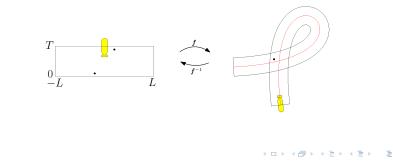




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Explored Area

- $W = [-L, L] \times [0, T]$,
- $\mathbb{A}_{\mathbb{E}} = \boldsymbol{f}(W)$,
- Sensor's Contour $\gamma = f(\partial W)$.



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Coverage Measure

$$c_m(\boldsymbol{p}) = \# Ker(\boldsymbol{f} - \boldsymbol{p})$$







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Coverage Measure

$$c_m(\boldsymbol{p}) = \# Ker(\boldsymbol{f} - \boldsymbol{p})$$



$$\mathbb{A}_{\mathbb{E}} = \{oldsymbol{p} \in \mathbb{R}^2 | c_m(oldsymbol{p}) \geq 1\}$$





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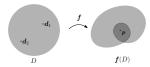




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Topological Degree

- D is an open subset of \mathbb{R}^n ,
- $\boldsymbol{f}:\overline{D}\to\mathbb{R}^n$ is continuous,
- $\boldsymbol{p} \in \mathbb{R}^n \backslash \boldsymbol{f}(\partial D)$
- deg : $(\boldsymbol{f}, D, \boldsymbol{p}) \rightarrow \mathbb{Z}$.



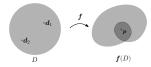




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Topological Degree

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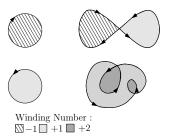
If det(f'(d)) is non-zero on each d such that f(d) = p,

$$deg(\boldsymbol{f}, D, \boldsymbol{p}) = \sum_{\boldsymbol{d} \in \boldsymbol{f}^{-1}(\boldsymbol{p})} sign(det(\boldsymbol{f}'(\boldsymbol{d})))$$





Winding Number



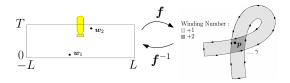
- *D* is an open subset of \mathbb{R}^2 ,
- $\boldsymbol{f}:\overline{D}
 ightarrow\mathbb{R}^2$ is continuous,
- $\boldsymbol{p} \in \mathbb{R}^2 \setminus \boldsymbol{f}(\partial D)$,
- $\eta(\boldsymbol{f}(\partial D), \boldsymbol{p}) \in \mathbb{Z}.$





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For any
$$\boldsymbol{p} \in \mathbb{R}^2$$
, $c_m(\boldsymbol{p}) = \eta(\gamma, \boldsymbol{p})$



If det(f'(w)) is positive on each $w \in W$ such that f(w) = p,

$$\eta(\gamma, \boldsymbol{p}) = \sum_{\boldsymbol{w} \in \boldsymbol{f}^{-1}(\boldsymbol{p})} \textit{sign}(\textit{det}(\boldsymbol{f}'(\boldsymbol{w}))) = \#\textit{Ker}(\boldsymbol{f} - \boldsymbol{p})$$



Costa Vianna M.L., Goubault E., Jaulin L., Putot S. (2022). Estimating the Coverage Measure and the Area Explored by a Side-Scan Sonar. *OCEANS 2022*

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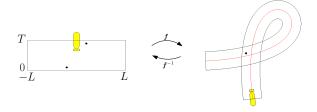
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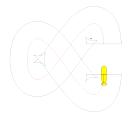
If det(f'(w)) is positive on each $w \in W$ such that f(w) = p,

$$c_m(\boldsymbol{p}) = \eta(\gamma, \boldsymbol{p})$$





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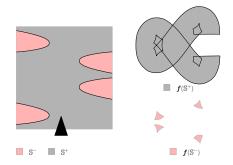


$$\eta(\gamma, \boldsymbol{p}) = \sum_{\boldsymbol{w} \in \boldsymbol{f}^{-1}(\boldsymbol{p})} sign(det(\boldsymbol{f}'(\boldsymbol{w}))) = +1 - 1 + 1 = +1 \neq \# Ker(\boldsymbol{f} - \boldsymbol{p})$$

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Sweeping backwards



$$\mathbb{S}^+ = \{ oldsymbol{w} \in W | det(oldsymbol{f}'(oldsymbol{w})) > 0) \}, \ \gamma^+ = oldsymbol{f}(\partial \mathbb{S}^+)$$

 $\mathbb{S}^- = \{ oldsymbol{w} \in W | det(oldsymbol{f}'(oldsymbol{w})) < 0) \}, \ \gamma^- = oldsymbol{f}(\partial \mathbb{S}^-)$



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$$c_m(oldsymbol{p}) = \# {\it Ker}(oldsymbol{f}-oldsymbol{p}) = \# {\it Ker}\ (oldsymbol{f}-oldsymbol{p})_{|\mathbb{S}^+} + \# {\it Ker}\ (oldsymbol{f}-oldsymbol{p})_{|\mathbb{S}^-}$$

$$c_m(\boldsymbol{p}) = \sum_{\boldsymbol{w} \in \boldsymbol{f}_{|\mathbb{S}^+}^{-1}(\boldsymbol{p})} + 1 + \sum_{\boldsymbol{w} \in \boldsymbol{f}_{|\mathbb{S}^-}^{-1}(\boldsymbol{p})} + 1 = \eta(\gamma^+, \boldsymbol{p}) + \eta(\gamma^-, \boldsymbol{p})$$





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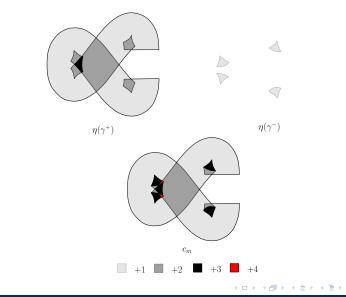
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Data

- DVL,
- IMU,
- Pressure.

Mission

- Classical survey path (law-mowing pattern),
- Roadstead of Brest (France, Brittany),
- 47 minutes.







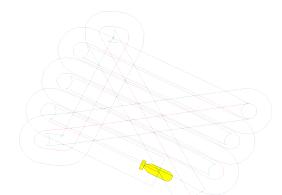
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¹S. Rohou, B. Desrochers, et al., The Codac library – Constraint-programming for robotics, 2022

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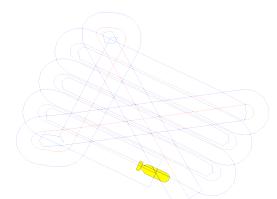




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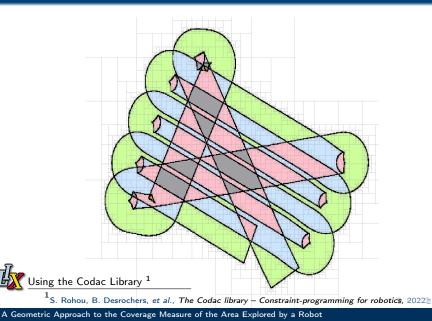
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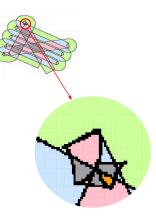
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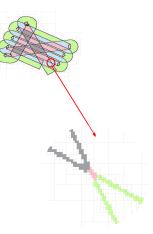
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- Relation between topological degree and exploration problem,
- coverage measure,
- algorithm for area characterization in terms of winding number.





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Future W	/ork				

Extensions of the current method:

- Patch Exploration,
- uncertainty in the robot's trajectory using thick sets.



